Are vortices in rotating superfluids breaking the Weak Equivalence Principle?

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September 16, 2009

Abstract

Due to the breaking of gauge symmetry in rotating superfluid Helium, the inertial mass of a vortex diverges with the vortex size. The vortex inertial mass is thus much higher than the classical inertial mass of the vortex core. An equal increase of the vortex gravitational mass is questioned. The possibility that the vortices in a rotating superfluid could break the weak equivalence principle in relation with a variable speed of light in the superfluid vacuum is debated. Experiments to test this possibility are investigated on the bases that superfluid Helium vortices would not fall, under the single influence of a uniform gravitational field, at the same rate as the rest of the superfluid Helium mass.

1 Introduction

Vortex dynamics was closely studied in a variety of materials called quantum liquids and solids [4], including Bose Einstein Condensates, Type-II superconductors and in rotating superfluid ^4He. Here we will concentrate on the latter system. The vortex inertial mass in superfluid ^4He, has been extensively discussed by Duan, Thouless and Popov in [1][2][3]. In contrast the discussion of the vortex gravitational mass is particularly poor. To the knowledge of the author, the validity of the weak equivalence principle for vortices in rotating superfluids has not been tested so far. The theoretical discussion of this subject is also scarce since presently the weak equivalence principle cannot be deduced on a purely theoretical base.

The Weak Equivalence Principle is one of the main foundations of the theory of general relativity. It means the constancy of the ratio between the inertial and the gravitational mass m_i and m_g respectively of a given physical system.

$$\frac{m_g}{m_i} = \iota = Cte \tag{1}$$

This implies, in classical physics, that the possible motions in a gravitational field are the same for different test particles. Current experimental tests of the weak equivalence principle [5] [6], indicate that the gravitational and inertial masses of any classical physical system should be equal to each other, $m_g/m_i = \iota = 1$, within a relative accuracy of the Eötvös-factor, $\eta(A,B)$ less than 5×10^{-13}.

$$\eta(A,B) = (m_g/m_i)_A - (m_g/m_i)_B < 5 \times 10^{-13} \tag{2}$$

However, it should be stressed that until now, all the experimental tests of the weak equivalence principle have been carried out with physical systems that do not break gauge invariance, contrary to the superfluids that break this symmetry. The Eötvös-factor is usually obtained from the measurement of the differential acceleration, Δa, of two free falling test bodies, A and B.

$$\eta(A,B) = \frac{\Delta a}{g_0} \tag{3}$$

where g_0 is the Earth's gravitational acceleration.

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In this paper we argue that *due to gauge symmetry breaking, the weak equivalence principle is violated by superfluid vortices*.

The paper starts with the presentation of the current understanding of superfluid vortex inertial mass. In section 3 a variable effective Lorentzian vacuum speed of light in superfluids associated with a breaking of the weak equivalence principle for vortices is derived from the conservation of energy. In section 4 we discuss the possibility to measure a breaking of the weak equivalence principle in superfluids through free fall experiments with rotating superfluid samples. Finally we close with a discussion of several physical arguments supporting our central hypothesis.

2 Vortex inertial mass in rotating superfluids

A vortex line in rotating superfluid Helium 4 is a topological singularity, which consists of a normal core region of the size of the coherence length ξ, and an outside region of circulating supercurrent. The coherence length can be estimated from the Heisenberg uncertainty principle.

$$\xi \sim \frac{\hbar}{m_{He} c_s} \tag{4}$$

where m_{He} is the bare atomic mass in ^4He and c_s is the speed of sound in the superfluid. Taking $c_s \sim 2 \times 10^2 m/s$ [7] we estimate $\xi \sim 1\mathring{A}$. In the theoretical framework of the classical fluid model the only obvious contribution to the vortex mass is the core mass[8].

$$m_{core} = L\pi\xi^2\rho \tag{5}$$

where L is the length of the vortex line, and $\rho = Nm$ is the density with N the bulk number density of ^4He atoms. This small vortex mass is usually discarded in the equations of motion of vortex dynamics since it contradicts experimental data.

Duan in [1] shown that due to spontaneous gauge symmetry breaking in superfluids, the condensate compressibility contributes to a vortex mass which is much larger than the classical core mass. He calculates that the vortex inertial mass turns out to diverge logarithmically with the system size.

$$m_{inertial} = m_{core} \ln\left(\frac{L}{\xi}\right) \tag{6}$$

Where L is the length of the vortex. For a practical superfluid system $\ln(L/\xi) \sim 20 - 30$.

The number of vortices N_v appearing in a cylindrical sample of superfluide ^4He rotating with angular velocity Ω is deduced from the quantization of the vortex canonical momentum.

$$N_v = \frac{2\pi R^2 \Omega}{\hbar/m} \tag{7}$$

where R is the radius of the superfluid sample. The total increase of the inertial mass of a rotating superfluid sample with respect to the same non-rotating sample, is obtained from eq.(6) and eq.(7)

$$\Delta M_{inertial} = N_v m_{core}\Big(\ln\left(\frac{L}{\xi}\right) - 1\Big) \tag{8}$$

Assuming that the weak equivalence principle is still valid in superfluids this overall increase of inertial mass should appear together with a similar increase of the gravitational mass of the superfluid sample.

$$\Delta M_{inertial} = \Delta M_{gravitational} \tag{9}$$

Thus we should observe an increase of the weight of the rotating superfluid sample with respect to the same sample in the stationary state. Taking a cylindrical sample of radius $R = 1cm$ and $\ln(L/\xi) \sim 20 - 30$, rotating at $\Omega = 1Rad/s$ in eq.(8), we estimate that the total increase of gravitational mass is of the order of $\Delta M_{gravitational} = 10^{-14} - 10^{-9} Kg$. Thus the experimental detection of the associated increase of weight of the overall sample is a challenging task to perform, that has not yet been overcome by experimentalists. In summary Until the present date the weak equivalence principle has not been tested for superfluid vortices. Since the weak equivalence principle cannot be demonstrated on a purely theoretical basis, it can only be justified by experiment, and since it has only been experimentally investigated

for the case of physical systems that do not break gauge invariance, the assumption that the gravitational and the inertial mass of a rotating superfluid sample are equal, eq.(9), is an hypothesis that needs to be carefully investigated at theoretical and experimental level. Specially because a breaking of gauge symmetry also makes the superfluid sample a preferred frame in contradiction with the foundations of relativistic mechanics. Therefore it does not seem too outrages or exotic to question the hypothesis that *the increase of inertial mass of the rotating superfluid sample, due to gauge symmetry breaking, is appearing together with a corresponding increase of weight of the same sample*, eq.(9), and to envisage the physical consequences of a possible breaking of the weak equivalence principle for superfluid vortices.

3 Variable vacuum speed of light in superfluids and breaking of the weak equivalence principle for superfluid vortices

The breaking of gauge symmetry makes the superfluid sample a preferred frame, this should be associated with a speed of light in the superfluid vacuum different from its classical value c_0, appearing in Lorentz transformations. As demonstrated by Duan and Popov [1] [3] the vortex inertial mass can be expressed in function of the vortex static energy ϵ_0 which is also logarithmically divergent as the sample size.

$$m_{inertial} = \frac{\epsilon_0}{c_s^2} \tag{10}$$

where c_s is the speed of sound in the superfluid. Making eq.(6) equal to eq.(10) we conclude that the effective Lorentzian speed of light c_{eff} for a rotating superfluid sample is:

$$c_{eff} = c_s \Big(\ln \Big(\frac{L}{\xi} \Big) \Big)^{1/2} \tag{11}$$

For practical values $c_{eff} \sim 4c_s - 5c_s$. We stress that c_{eff} should be the value of the speed of light to take into account when carrying out coordinate transformations between a frame attached to the superfluid sample and a frame outside the superfluid sample.

Starting from Mach's principle, which asserts that there is a connection between the local laws of physics and the large scale properties of the universe, Sciama in [9] introduced the relation

$$c^2 = \frac{2GM}{R} \tag{12}$$

where R and M are the radius and the mass of the universe. Einstein's relationship linking energy and mass then takes the form

$$E = mc^2 = \frac{2GMm}{R} \tag{13}$$

which can be interpreted as a statement that the inertial energy that is present in any physical object is due to the gravitational potential energy of all the matter in the universe acting on the object. Therefore the mass m appearing in eq.(13) should be the gravitational mass of the object.

$$E = m_{gravitational}\ c_0^2 \tag{14}$$

Since the rest mass energy of the vortex ϵ_0 must be conserved independently of the value of the vacuum speed of light, the gravitational mass will adjust its value to compensate the variation of the speed of light in the superfluid vacuum.

$$m_{gravitational}\ c_0^2 = m_{core} c_s^2 \ln \Big(\frac{L}{\xi} \Big) \tag{15}$$

From eq.(15) we deduce that the gravitational mass of a superfluid vortex $m_{gravitational}$ is proportional to the classical vortex core mass and also diverges logarithmically as the size of the vortex.

$$m_{gravitational} = \Big(\frac{c_s}{c_0} \Big)^2 m_{core} \ln \Big(\frac{L}{\xi} \Big) \tag{16}$$

where the proportionality coefficient is equal to the square of the ratio between the speed of sound in the superfluid c_s and the classical speed of light in vacuum c_0. Comparing eq.(6) and eq.(16) we conclude that due to the principle of energy conservation and

to the breaking of guage invariance in superfluids the inertial and the gravitational mass of a vortex cannot be equal to each other. Therefore the weak equivalence principle should break for the case of superfluid vortices.

4 Rotating superfluids in free fall

As we have shown in section 2, measuring the vortices gravitational mass comparing the weight of the superfluid sample in rotating and stationary state is challenging due to the extremely small value of the vortex core mass. However in free fall experiments with rotating superfluid samples it should be possible to measure the differential acceleration Δa between the vortex and the bulk superfluid. The Eötvös factor η associated with the free fall of a vortex and the superfluid bulk under the single influence of the Earth gravitational field g_0 would be obtained from eq.(3):

$$\eta = \frac{\Delta a}{g_0} \tag{17}$$

Let us assume that the friction force between the vortex and the superfluid bulk is null. On one side, since the superfluid bulk inertial and gravitational mass are equal, the center of mass of the superfluid bulk will fall with and acceleration

$$a_{superfluid} = g_0 \tag{18}$$

On the other side The vortex will fall according to the equation of motion

$$g_0 \; m_{gravitational} = m_{inertial} \; a_{vortex} \tag{19}$$

substituting eq.(16), and eq.(6) in eq.(19) we calculate the vortex falling acceleration

$$a_{vortex} = g_0 \frac{c_s}{c} \tag{20}$$

Substituting the accelerations $a_{superfluid}$, eq(18), and a_{vortex}, eq.(20), in eq.(3) we obtain the Eötvös factor η for a superfluid vortex with respect to the superfluid bulk.

$$\eta = 1 - \left(\frac{c_s}{c_0}\right)^2 \tag{21}$$

Taking $c_s \sim 2 \times 10^2 m/s$ we have $\eta \sim 1$ which is much higher than the upper limit measured for classical material systems of 5×10^{-13}, eq.(2). By comparing the equation of motion of the vortex and superfluid bulk we can easily show that the relative displacement between the vortex center of mass and the superfluid bulk center of mass is equal to the Eötvös factor η

$$\frac{\Delta d}{d} = \eta \tag{22}$$

where d is the distance covered by the superfluid bulk center of mass after a free fall time t. Since the ceiling of the cylindrical container, enclosing the superfluid, will not allow the vortex to escape the container we deduce that the vortex length will shrink while the rotating container is in free fall. In the most optimistic case the vortex will only disappear when the container has fall a distance comparable to half its total length $d = L/2$(the vortex would then be flatten to the ceiling of the rotating container). At this instant the vortex angular momentum would be restituted to the rotating container, followed by the creation of a new vortex with length L. Therefore the rotating container would exhibit a total angular momentum varying in time with a period $T = \sqrt{L/g_0}$, equal to the period of a classical pendulum with length L, and vortices would be created an annihilated with the same period of time. In the case where the weak equivalence principle is preserved superfluid vortices and the superfluid container angular momentum should not be affected by the free fall.

If instead of assuming no friction between the vortices and the superfluid bulk, like we did above, we assume an ideal rigid connection between both systems. We deduce from the equation of motion of the freely falling rotating superfluid sample, a falling acceleration a_z.

$$a_z = \frac{1 + \left(\frac{c_s}{c}\right)^2 \frac{m_v}{m}}{1 + \frac{m_v}{m}} g_0 \tag{23}$$

where m is the total classical mass of the superfluid bulk (without the vortices) and $m_v = N_v m_{core} \ln\left(\frac{L}{\xi}\right)$ is the total inertial mass of vortices

in the superfluid sample, with N_v being the total number of vortices. Comparing this acceleration with the falling acceleration of the same non-rotating sample, g_0, we calculate the Eötvös factor η' of the rotating sample with respect to the non-rotating one.

$$\eta' = \frac{g_0 - a_z}{g_0} \tag{24}$$

substituting eq.(23) in eq.(24) we obtain

$$\eta' = \frac{m_v}{\Delta m}\eta \tag{25}$$

where $\Delta m = m - m_v$ and $\eta = 1 - \left(\frac{c_s}{c_0}\right)^2$ is the Eötvös factor of one vortex with respect to the superfluid bulk (assuming no friction between both), eq.(21). Taking a cylindrical sample of radius $R = 1cm$ and $\ln(L/\xi) \sim 20-30$, rotating at $\Omega = 1Rad/s$ in eq.(25), we estimate the order of magnitude of $\eta' \sim 10^{-11}$, which is 2 orders of magnitude above the upper limit experimentally determined for normal materials, which do not break gauge invariance, eq.(2). From eq.(22), which also applies in the present free fall experiment, $\eta' = \Delta d/d$, with $\eta' \sim 10^{-11}$, we deduce that large free fall distances d are required to clearly measure this effect, i. e. to measure a detectable Δd between the distance covered by the rotating and the non-rotating sample. This could actually be achieved by conducting the experiment on board a satellite in free fall around the Earth.

5 Discussion and Conclusions

In [2] Thouless and Anglin shown how to obtain an expression for the inertial mass of a stable quantized vortex in an infinite neutral superfluid by subjecting it to a straight, circularly symmetric, pinning potential which is slowly and steadily rotated about an axis parallel to the vortex line whose distance from the vortex is large compared with the size of the vortex. They find that the vortex mass depends strongly on the pinning potential, and diverges when its radius tends to zero. If we consider an hypothetical gravitational pinning potential generated by an adequate distribution of mass, we would thus reach the conclusion that the vortex inertial mass would depend in a divergent manner on the gravitational mass of the source of the pinning potential. Therefore in general the vortex inertial mass would not cancel out in the equation of motion of the vortex relative to the pinning potential. This represents an additional argument in favor of breaking of the weak equivalence principle for superfluid vortices.

In analogy with the present discussion for superfluids, spontaneous breaking of gauge invariance in superconductors could also lead to a breaking of the weak equivalence principle for Cooper pairs [10]. In this case also a vacuum Lorentz speed of light different from the classical one, is needed to preserve energy conservation of the Cooper pairs rest-mass energy. The anomalous Cooper pair inertial mass excess reported by Tate et al. [11] would thus be the analog in superconductors of the diverging inertial mass of a vortex in superfluids.

More generally, as widely discussed in the literature [12] the validity of the weak equivalence principle in the framework of quantum mechanics is not at all granted. Since superconductivity and superfluidity are macroscopic quantum effects, it should not be too surprising to find some anomalies with the weak equivalence principle in these systems. Ultimately, it seems to the author, that this subject should find a general solution in the larger problem of the correct physical interpretation of quantum and relativistic mechanics. Meaning that if the Copenhagen interpretation of quantum mechanics is correct we should indeed expect a breaking of the weak equivalence principle in physical quantum systems.

From the analysis presented in section 2 and 4, we see that due to the small value of the vortex core mass, which is in the order of $m_{core} \sim 10^{-20} - 10^{-15} Kg$ (taking $\ln(L/\xi) \sim 20 - 30$), and due to a speed of sound in the superfluid which is much lower than the classical speed of light in vacuum, $c_s \ll c_0$, a possible breaking of the weak equivalence principle for vortices would not be easy to detect either through weighting measurements of the rotating and non rotating sample or through the measurement of anomalous free fall delay times between two superfluid samples one being rotating, the other being stationary. The most accessible physical parameters to investigate a possible breaking of the weak equiva-

lence principle for superfluid vortices would be the monitoring of the superfluid container angular momentum, together with the area density of vortices, while the rotating sample is in free fall. An harmonic oscillation of this parameters with period $T = \sqrt{L/g_0}$ should be observed.

To conclude, it seems that the current theoretical understanding of superfluid vortex inertial mass resulting from a breaking of gauge invariance in superfluids, justifies a careful investigation of the validity of the weak equivalence principle for superfluid vortices in the context of free fall experiments with rotating superfluid samples carried out on Earth or in space.

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